MODEL ANSWER

Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Decision Mathematics 2

Sample Assessment Material for first teaching September 2017
Time: 50 minutes

Paper Reference
8FM0/2K

You must have:
Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If a pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.
- Do not return this question paper with the answer book.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 4 questions in this question paper. The total mark for this paper is 40.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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S 5 8 5 3 6 A 0 1 1 1

Answer ALL questions. Write your answers in the answer book provided.

1. Six workers, A, B, C, D, E and F, are to be assigned to five tasks, P, Q, R, S and T.

Each worker can be assigned to at most one task and each task must be done by just one worker.

The time, in minutes, that each worker takes to complete each task is shown in the table below.

	P	Q	R	S	T
A	32	32	35	34	33
B	28	35	31	37	40
C	35	29	33	36	35
D	36	30	34	33	35
E	30	31	29	37	36
F	29	28	32	31	34

Reducing rows first, use the Hungarian algorithm to obtain an allocation which minimises the total time. You must explain your method and show the table after each stage.

(Total for Question 1 is 9 marks)

1)

Dummy variable X

	P	Q	R	S	T	X	Row reduce by
A	32	32	35	34	33	40	32
B	28	35	31	37	40	40	28
C	35	29	33	36	35	40	29
D	36	30	34	33	35	40	30
E	30	31	29	37	36	40	29
F	29	28	32	31	34	40	28

	P	Q	R	S	T	X
A	0	0	3	2	1	8
B	0	7	3	9	12	12
C	6	0	4	7	6	11
D	6	0	4	3	5	10
E	1	2	0	8	7	11
F	1	0	4	3	6	12

Column reduce by

0 0 0 2 1 8

$\longrightarrow$

	P	Q	R	S	T	X
A	0	0	3	0	0	0
B	0	7	3	7	11	4
C	6	0	4	5	5	3
D	6	0	4	1	4	2
E	1	2	0	6	6	3
F	1	0	4	1	5	4

	P	Q	R	S	T	X
A	0	0	3	0	0	0
B	0	7	3	7	11	4
C	6	0	4	5	5	3
D	6	0	4	1	4	2
E	1	2	0	6	6	3
F	1	0	4	1	5	4

minimum uncovered is 1

Augmented by 1

	P	Q	R	S	T	X
A	1	1	3	0	0	0
B	0	7	2	6	10	3
C	6	0	3	4	4	2
D	6	0	3	0	3	1
E	2	3	0	6	6	3
F	1	0	3	0	4	3

$\longrightarrow$

	P	Q	R	S	T	X
A	1	1	3	0	0	0
B	0	7	2	6	10	3
C	6	0	3	4	4	2
D	6	0	3	0	3	1
E	2	3	0	6	6	3
F	1	0	3	0	4	3

smallest uncovered is 1

Augmented by 1

	P	Q	R	S	T	X
A	2	2	3	1	0	0
B	0	7	1	6	9	2
C	6	0	2	4	3	1
D	6	0	2	0	2	0
E	3	4	0	7	6	3
F	1	0	2	0	3	2

$\longrightarrow$

	P	Q	R	S	T	X
A	2	2	3	1	0	0
B	0	7	1	6	9	2
C	6	0	2	4	3	1
D	6	0	2	0	2	0
E	3	4	0	7	6	3
F	1	0	2	0	3	2

6 lines $\Rightarrow$ solution is optimal

A T
B P
C Q
D (X)
E R
F S

2. In two-dimensional space, lines divide a plane into a number of different regions.

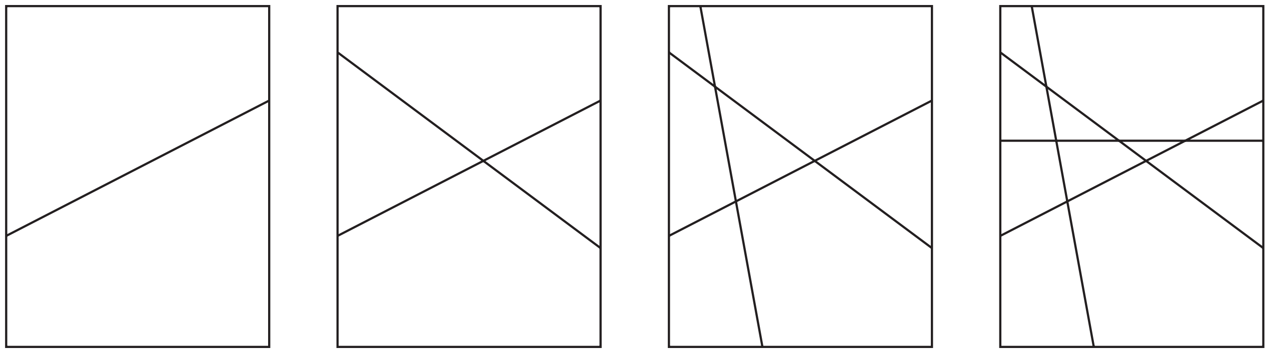


Figure 1 **Figure 2** **Figure 3** **Figure 4**

It is known that:

- One line divides a plane into 2 regions, as shown in Figure 1
- Two lines divide a plane into a maximum of 4 regions, as shown in Figure 2
- Three lines divide a plane into a maximum of 7 regions, as shown in Figure 3
- Four lines divide a plane into a maximum of 11 regions, as shown in Figure 4

(a) Complete the table in the answer book to show the maximum number of regions when five, six and seven lines divide a plane.

(1)

(b) Find, in terms of u_n, the recurrence relation for u_{n+1}, the maximum number of regions when a plane is divided by $(n + 1)$ lines where $n \geqslant 1$

(1)

(c) (i) Solve the recurrence relation for u_n

(ii) Hence determine the maximum number of regions created when 200 lines divide a plane.

(3)

(Total for Question 2 is 5 marks)

(2a)

no. of lines	1	2	3	4	5	6	7
no. of regions	2	4	7	11	16	22	29

Spot the pattern from (a)

(2b) $u_{n+1} = u_n + n + 1$

(2c) let $p(n) = n+1$ so $u_{n+1} = u_n + p(n)$

$\Rightarrow u_n = u_{n+1} + p(n)$

$\Rightarrow u_n = \lambda n^2 + \underbrace{\mu n + \phi}_{= p(n)}$ form of partical solution

when $n=1$, $u_n = 2$

$2 = \lambda + \mu + \phi$ (1)

when $n=2$, $u_n = 4$

$4 = 4\lambda + 2\mu + \phi$ (2)

when $n=3$, $u_n = 7$

$7 = 9\lambda + 3\mu + \phi$ (3)

Solve simultaneously to find values of λ, μ, ϕ

(2)−(1): $4 = 4\lambda + 2\mu + \phi$

$2 = \lambda + \mu + \phi$

$2 = 3\lambda + \mu \iff \mu = 2 - 3\lambda$

sub into (3)

$7 = 9\lambda + 3(2 - 3\lambda) + \phi$

$\iff 7 = 9\lambda + 6 - 9\lambda + \phi$

$\iff \underline{\phi = 1}$

sub into (1): $2 = \lambda + \mu + 1 \Rightarrow \lambda + \mu = 1 \Rightarrow \lambda = 1 - \mu$

Sub $\lambda = 1 - \mu$ into (2)

$4 = 4(1-\mu) + 2\mu + 1$

$\Rightarrow 3 = 4 - 4\mu + 2\mu$

$\Rightarrow -1 = -2\mu$

$\Rightarrow \underline{\mu = \frac{1}{2}}$

$\lambda = 1 - \mu$

$= 1 - \frac{1}{2}$

$\underline{= \frac{1}{2}}$

so

$u_n = \lambda n^2 + \mu n + \phi$

$= \frac{1}{2}n^2 + \frac{1}{2}n + 1$

$= \frac{1}{2}n(n+1) + 1$

3.

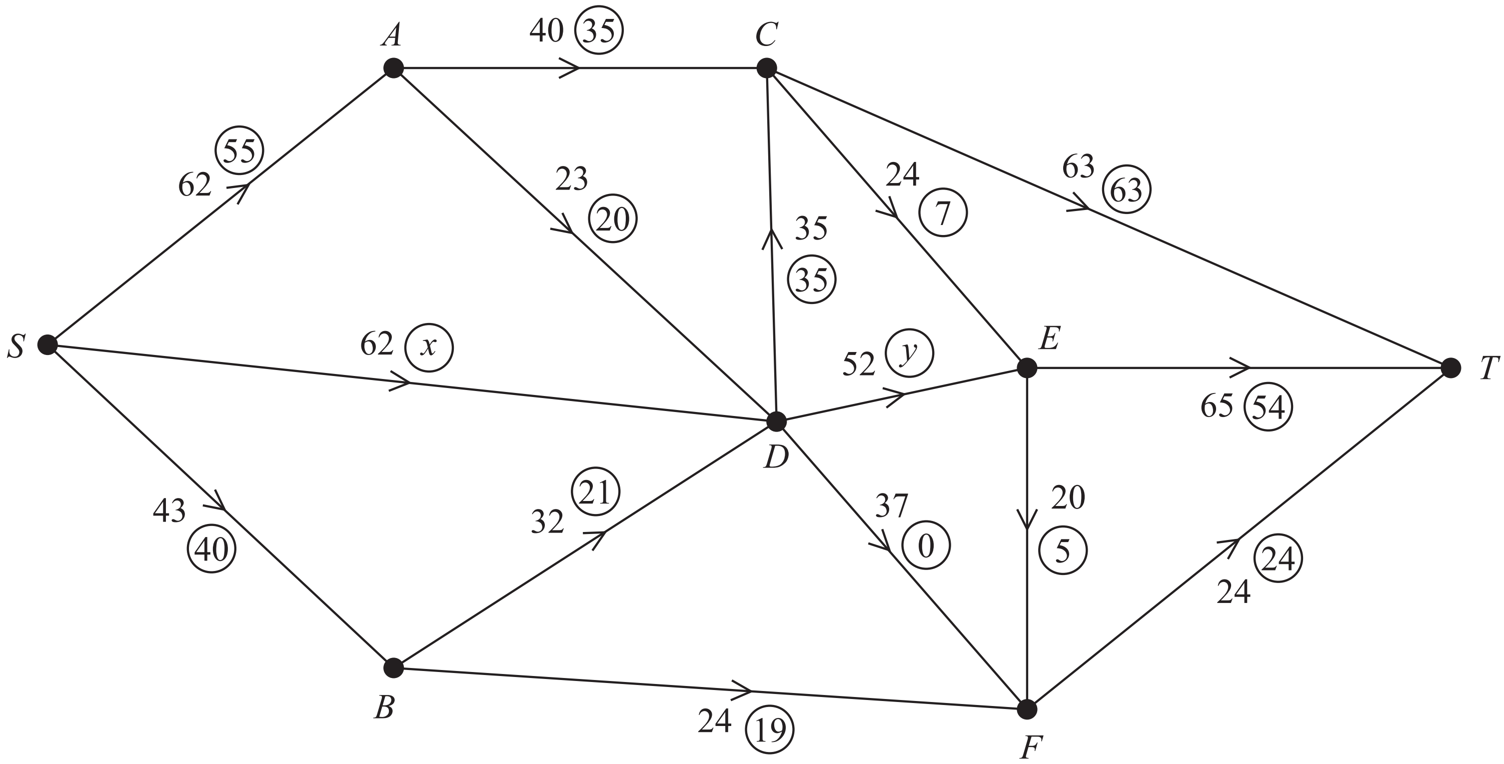


Figure 5

Figure 5 represents a network of corridors in a school. The number on each arc represents the maximum number of students, per minute, that may pass along each corridor at any one time. At 11 am on Friday morning, all students leave the hall (S) after assembly and travel to the cybercafé (T). The numbers in circles represent the initial flow of students recorded at 11 am one Friday.

(a) State an assumption that has been made about the corridors in order for this situation to be modelled by a directed network.

(1)

(b) Find the value of x and the value of y, explaining your reasoning.

(3)

Five new students also attend the assembly in the hall the following Friday. They too need to travel to the cybercafé at 11 am. They wish to travel together so that they do not get lost. You may assume that the initial flow of students through the network is the same as that shown in Figure 5 above.

(c) (i) List all the flow augmenting routes from S to T that increase the flow by at least 5

(ii) State which route the new students should take, giving a reason for your answer.

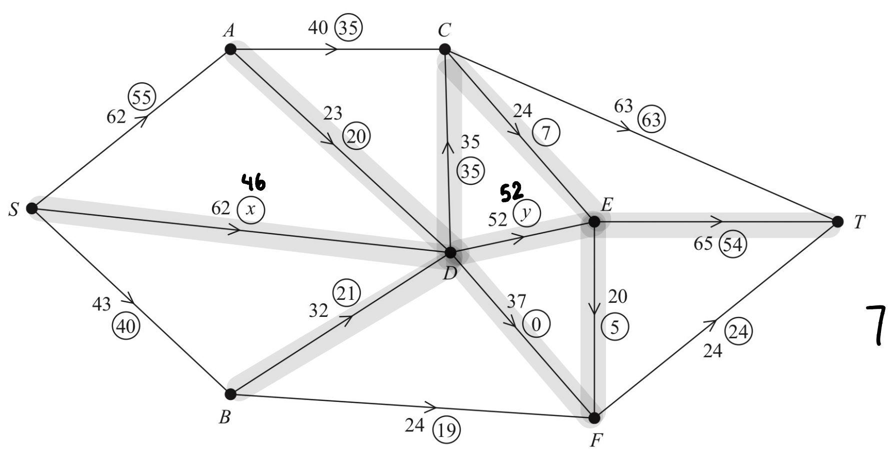
(3)

(d) Use the answer to part (c) to find a maximum flow pattern for this network and draw it on Diagram 1 in the answer book.

(1)

(e) Prove that the answer to part (d) is optimal.

(3)



a) Corridors must be one way

b) Input = output

For E :

$7 + y = 54 + 5 \Rightarrow y = 52$

For D :

$20 + 21 + x = 35 + y + \emptyset \Rightarrow x = 35 + 52 - 41 = 46$

c) i) SACET = 5

SDFET = 5

ii) can't use SDFET as E → F is one way so cannot use F → E

(d)

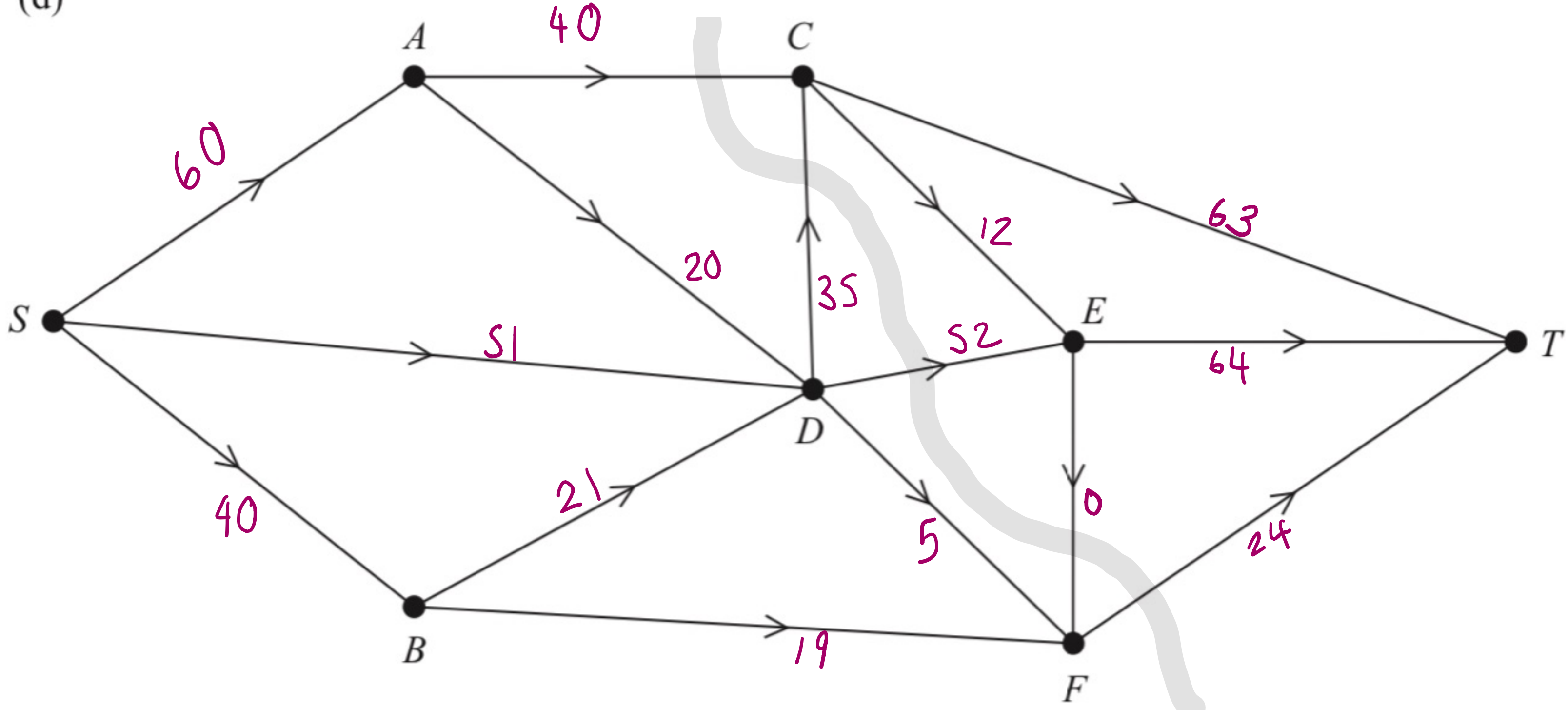


Diagram 1

e) max flow min cut theory

$40 + 35 + 52 + 24 = 151$

max flow $= 60 + 51 + 40 = 151$

These are equal so the flow is optimal

The school is intending to increase the number of students it takes but has been informed it cannot do so until it improves the flow of students at peak times. The school can widen corridors to increase their capacity, but can only afford to widen one corridor in the coming term.

(f) State, explaining your reasoning,

(i) which corridor they should widen,

(ii) the resulting increase of flow through the network.

(3)

(Total for Question 3 is 14 marks)

f) i) max flow into T is 152 so increasing any arc other than FT will only cause an increase of 1, so FT is the arc that should have the capacity increased.

ii) SAD, SD and SBD would be able to use max flow directed through DF and could increase by 16.

4. A two person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	**B plays 2**	**B plays 3**	min
A plays 1	4	1	2	1
A plays 2	2	4	3	2 max
max	4	4	3 min	

(a) Verify that there is no stable solution.

(3)

(b) (i) Find the best strategy for player A.

(ii) Find the value of the game to her.

(9)

(Total for Question 4 is 12 marks)

TOTAL IS 40 MARKS

a) Row minima 1,2 max = 2

Column maxima 4,3 min = 3

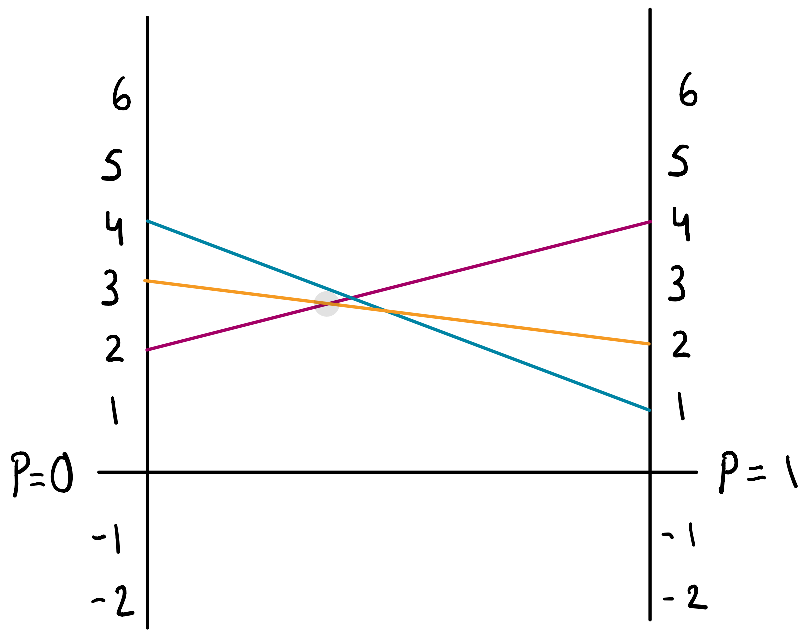
row minima 2 $\neq$ column maxima 3 so not stable

b) i) Let A play strategy 1 with probability p and strategy 2 with probability $1-p$

If B plays strategy 1 A's gains are :
$4(p) + 2(1-p) = 2p+2$

If B plays strategy 2 A's gains are :
$(p) + 4(1-p) = 4-3p$

If B plays strategy 3 A's gains are :
$2(p) + 3(1-p) = 3-p$



$2p+2 = 3-p$

$3p = 1 \Rightarrow p = \frac{1}{3}$

Player A should play strategy 1 $\frac{1}{3}$ of the time and strategy 2 $\frac{2}{3}$ of the time

ii) $3-p \Rightarrow 3-\frac{1}{3} = 2\frac{2}{3}$

The value of the game to player A is $2\frac{2}{3}$